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THE HORIZONTAL WIRE ANTENNA OVER A CONDUCTING OR DIELECTRIC HALF-SPACE: CURRENT AND ADMITTANCE

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-The wave number, the distribution of current, and the admittance of a horizontal wire antenna over a homogeneous isotropic half-space are determined when the complex wave number characteristic of the half-space is large in magnitude compared with the real wave number of the air. The axial electric field at the surface of the half-space is described. The properties of the antenna are obtained from those of the eccentrically insulated dipole in a general medium in the limit as the radius of the insulation is increased without

limit while the distance of the dipole from the boundary is kept constant. Application of the new theory to the Beverage or wave antenna is discussed.

1. INTRODUCTION

When a center-driven dipole antenna is placed parallel to a conducting or dielectric half-space at a height d above it, distributions of current and charge are induced in it that interact with the currents and charges in the antenna and combine with these to generate the electromagnetic field. The calculation of the interaction is difficult except when the half-space is perfectly conducting. In this case the effect of the distributions of current and charge on the conducting surface is precisely that of the equal and opposite currents and charges in a fictitious "image" antenna located at a distance d below the surface. It replaces the entire half-space and reduces the determination of currents, charges and electromagnetic fields to that of two identical parallel antennas separated by a distance 2d and driven by equal and opposite generators. When $2k_0d << 1$, the antenna and its image constitute a two wire transmission line with currents and driving-point admittance that are determined primarily by the impedance and capacitance per unit length of the line and affected only negligibly by radiation. This is true of the several circuits shown in Fig. 1: the center-driven line with open ends shown in Fig. 1a; the line terminated at each end in an arbitrary shunt impedance $Z_L/2$, where Z_L may be the characteristic impedance of the line, as in Fig. 1b; or the line with a series impedance $Z_L/2$ at a quarter wavelength from the open end as in Fig. 1c. In each case the image conductors and impedances are shown in broken lines. Note that the circuits of Figs. 1b and 1c are equivalent insofar as the current and driving-point admittance of the main line between the impedances $Z_L/2$ are concerned.

When the half-space below the center-driven dipole is not perfectly conducting but consists of a material like the earth, the sea or a fresh water lake, conduction or polarization currents or both of these may be induced. However, they are not confined to a thin surface layer but penetrate more or less deeply into the medium. Their effect on the current in the antenna and their contribution to the electromagnetic field are not easily determined. In the treatment of antennas parallel to the earth it has been common practice to assume the distribution of current to be known and to be substantially that for the same antennas when isolated [1]. This implicitly also takes for granted that the wave number for the current on the highly conducting antenna is the same as that for the isolated antenna, which is the free-space wave number $k_0 = \omega/c$ where c is the velocity of light. Perhaps the most im-

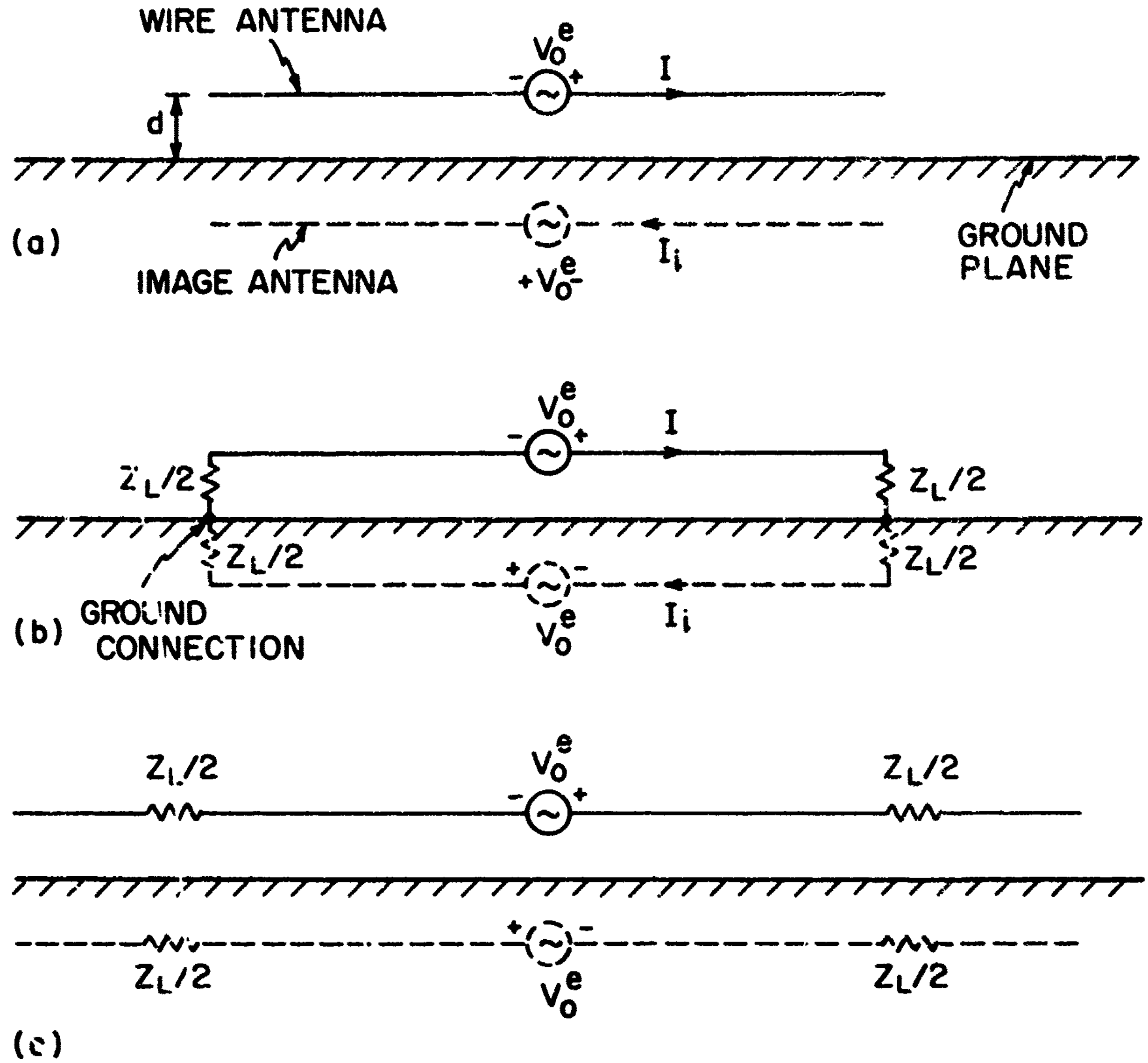

FIG. 1 HORIZONTAL WIRES OVER GROUND PLANE. BROKEN LINES SHOW IMAGE ANTENNA FOR PERFECT CONDUCTOR.

portant antenna of this type is the Beverage antenna which consists of a wire parallel to and close to the earth terminated in a manner to provide a traveling wave of current. This antenna was first described by Beverage [2]. It has been discussed by numerous authors primarily as a directional receiving antenna [3]-[6]. In general, a simple traveling wave with the free-space wave number has been assumed and the dependence of the wave number and of the distribution of current on the properties of the adjacent half-space has not been determined. Wait [7] derived a modal equation for the wave number for an assumed traveling wave along an infinitely long wire parallel to a dissipative half-space, but concluded that its solution in the general case is a "formidable task" and that "in spite of the fundamental nature of the problem, there has not been a great deal of progress in obtaining useful results." Even in the limiting transmission-line form, Wait expresses his final formula for the wave number in terms of an unevaluated integral.

2. REVIEW OF SOLUTION FOR CURRENT IN ECCENTRICALLY INSULATED ANTENNAS

In a recent paper [8] a new solution was derived for the current in and the admittance of a center-driven insulated dipole when immersed in an ambient medium with a characteristic complex wave number much greater in magnitude than that of the insulating material. This theory was verified by very extensive measurements of distributions of current and charge along air-insulated antennas immersed in a large lake [9]. The theory of the insulated antenna has now been generalized to include eccentric insulators [10]. That is, the antenna is not located along the axis of the circular cylinder of insulation but is displaced by an arbitrary distance D. The cross section of an eccentrically insulated antenna is shown in Fig. 2. The radius of the insulation is b, that of the conductor is a. The axis of the conductor is at a distance D from the axis of the insulator or a distance $d = b - D$ from its surface. The wave number characteristic of the insulating material in the range $a < r < b$ is $k_2 = \beta_2 + i\alpha_2 = \omega[\mu(\epsilon_2 + i\sigma_2/\omega)]^{1/2} = \omega(\mu\tilde{\epsilon}_2)^{1/2}$; the wave number characteristic of the ambient medium in the range $b < r < \infty$ is $k_4 = \beta_4 + i\alpha_4 = \omega[\mu(\epsilon_4 + i\sigma_4/\omega)]^{1/2} = \omega(\mu\tilde{\epsilon}_4)^{1/2}$ where β_4 is the real phase constant, α_4 the real attenuation constant. It is assumed in the analysis that the following two conditions are satisfied:

$$|k_4| >> |k_2| \quad ; \quad |k_2 d| << 1 \tag{1}$$

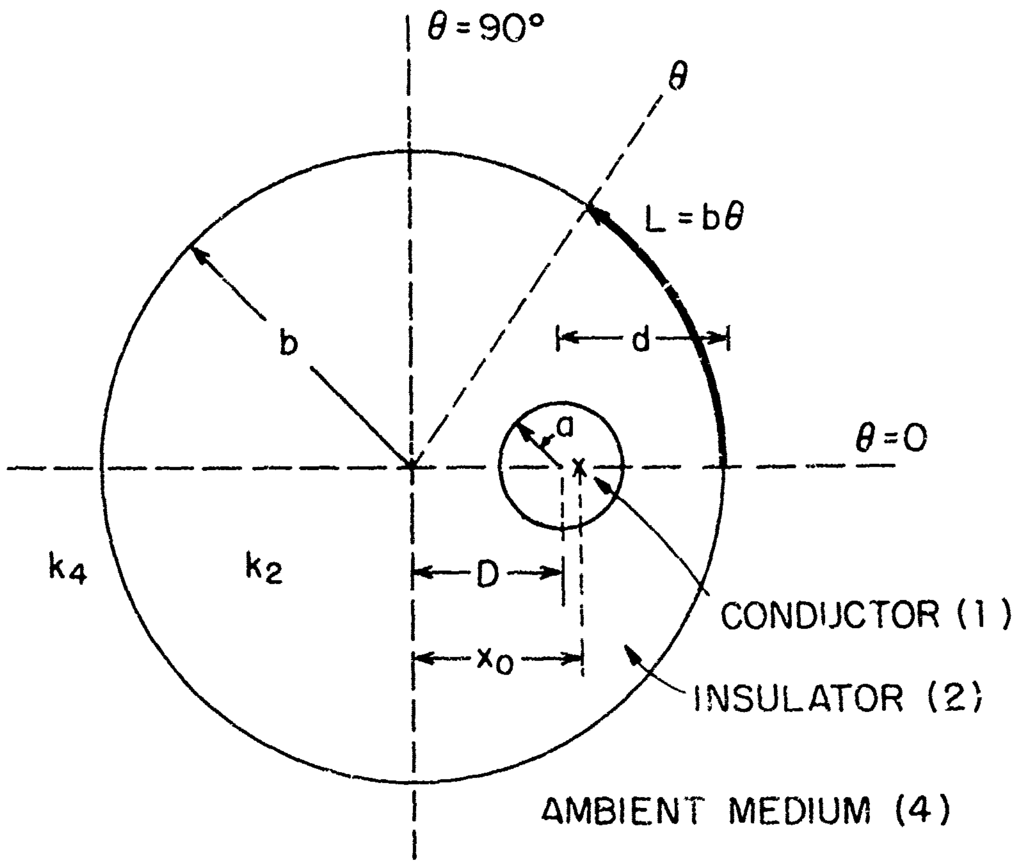

FIG. 2 CROSS SECTION OF ECCENTRICALLY INSULATED ANTENNA

Subject to these conditions the distribution of current along the eccentrically insulated dipole of half length h is

$$I(z) = \frac{iV_0^e \sin k_L(h - |z|)}{2Z_c \cos k_L h} \tag{2}$$

where a time dependence $\exp(-i\omega t)$ has been assumed and

$$Z_c = \frac{k_L \Omega_a}{2\pi\omega\tilde{\epsilon}_2} = \frac{\zeta_2 k_L \Omega_a}{2\pi k_2} \tag{3}$$

$$k_L = k_2\left[1 + \frac{\Delta_0}{(k_4 b)\Omega_a}\right]^{1/2} = \beta_L + i\alpha_L \tag{4}$$

$$\Omega_a = \cosh^{-1}\left(\frac{a^2 + b^2 - D^2}{2ab}\right) \tag{5}$$

$$\Delta_0 = \frac{H_0^{(1)}(k_4 b)}{H_1^{(1)}(k_4 b)} + 2\sum_{m=1}^{\infty}\left(\frac{Dx_0}{b^2}\right)^m \frac{H_m^{(1)}(k_4 b)}{H_{m+1}^{(1)}(k_4 b)} \tag{6}$$

The complex wave impedance of region 2 is $\zeta_2 = (\mu/\tilde{\epsilon}_2)^{1/2}$. In the last expression,

$$x_0 = \frac{1}{2D}\quad (b^2 + D^2 - a^2) - \sqrt{(b^2 + D^2 - a^2)^2 - 4b^2D^2} \tag{7}$$

This locates an infinitely thin wire as shown in Fig. 2.

The driving-point admittance is

$$Y = G - iB = \frac{I(0)}{V} = \frac{i}{2Z_c}\tan k_L h \tag{8}$$

This is the admittance of two open-ended sections of transmission line with characteristic impedance Z_c and complex wave number k_L in series with two generators each with driving voltage $V_0^e/2$ at $z = 0$ as indicated in Fig. 3.

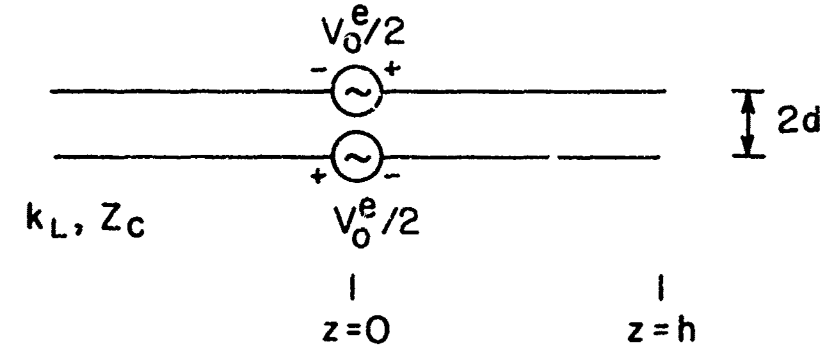

FIG. 3 TWO OPEN-ENDED SECTIONS OF LINE EACH OF LENGTH h IN SERIES WITH TWO GENERATORS WITH EMF's $V_0^e/2$.

3. THE COMPLEX WAVE NUMBER FOR THE ANTENNA NEAR A PLANE EARTH

The wave number for the antenna eccentrically placed in a circular cylinder of radius b is given by (4). In order to obtain its value in the limit $b \to \infty$, $D \to \infty$ with $d = b - D$ fixed and finite, it is necessary to determine Ω_a and Δ_0. The former is readily done. Thus,

$$\Omega_a = \cosh^{-1}\left[\frac{a^2 + d(b + D)}{2ab}\right] \to \cosh^{-1}\frac{d}{a} \doteq \ln\frac{2d}{a} \tag{9}$$

where the logarithmic approximation is valid when $a^2 \ll d^2$. The evaluation of Δ_0 is more difficult. As a first step, note that with (7)

$$x_0 \to \frac{1}{2D}\,[b^2 + D^2 - b^2 + D^2] = D = b - d \tag{10}$$

so that (6) becomes

$$\Delta_0 = \frac{H_0^{(1)}(k_4 b)}{H_1^{(1)}(k_4 b)} + 2\sum_{m=1}^{\infty}\left(\frac{b - d}{b}\right)^{2m}\frac{H_m^{(1)}(k_4 b)}{H_{m+1}^{(1)}(k_4 b)} \tag{11}$$

or, in a form more convenient for later use,

$$\Delta_0 = \lim_{\substack{b\to\infty\\ D\to\infty}}\left\{\frac{H_0^{(1)}(k_4 b)}{H_1^{(1)}(k_4 b)} + 2\left(\frac{b - d}{b}\right)^{-2}\sum_{m=2}^{\infty}\left(\frac{b - d}{b}\right)^{2m}\frac{H_{m-1}^{(1)}(k_4 b)}{H_m^{(1)}(k_4 b)}\right\}$$

$$= \lim_{\substack{b\to\infty\\ D\to\infty}}\left\{ i + 2\sum_{m=2}^{\infty}\left(\frac{b - d}{b}\right)^{2m}\frac{H_{m-1}^{(1)}(k_4 b)}{H_m^{(1)}(k_4 b)}\right\} \tag{12}$$

It is not permissible to take the limit $b \to \infty$ before the evaluation of the sum, since this involves $m \to \infty$. This means that the familiar asymptotic forms for the Hankel functions of large argument cannot be used for $m > |k_4 b|$ since they are derived on the assumption that the order is small compared to the argument. They have been used for orders 0 and 1. What is required is an expression for the Hankel functions that is valid for both large orders and large arguments. Such a form may be obtained with the recurrence relation

$$zH_m^{(1)\prime}(z) + mH_m^{(1)}(z) = zH_{m-1}^{(1)}(z) \tag{13}$$

which yields

$$\frac{H_{m-1}^{(1)}(z)}{H_m^{(1)}(z)} = \frac{H_m^{(1)\prime}(z)}{H_m^{(1)}(z)} + \frac{m}{z} = \frac{d}{dz}\,\ell n[H_m^{(1)}(z)] + \frac{m}{z} \tag{14}$$

where the prime denotes the derivative with respect to the argument. The following results are obtained with formulas given in Bateman [11, p. 86]:

$$z > m: \quad \frac{d}{dz}\,\ell n[H_m^{(1)}(z)] = \frac{d}{dz}\,[-\frac{1}{4}\,\ell n(z^2 - m^2) + i(z^2 - m^2)^{1/2} + im\,\sin^{-1}(m/z)] \tag{15}$$

$$z < m: \quad \frac{d}{dz}\,\ell n[H_m^{(1)}(z)] = \frac{d}{dz}\,[-\frac{1}{4}\,\ell n(m^2 - z^2) - (m^2 - z^2)^{1/2} + m\,\cosh^{-1}(m/z)] \tag{16}$$

When the differentiation is carried out,

$$z > m: \quad \frac{d}{dz}\,\ell n[H_m^{(1)}(z)] = -\frac{1}{4}\frac{2z}{z^2 - m^2} + i\frac{z}{\sqrt{z^2 - m^2}} - i\frac{m^2}{z\sqrt{z^2 - m^2}}$$

$$\doteq \frac{i}{z}\sqrt{z^2 - m^2} \tag{17}$$

$$z < m: \quad \frac{d}{dz}\,\ell n[H_m^{(1)}(z)] = \frac{1}{4}\frac{2z}{m^2 - z^2} + \frac{z}{\sqrt{m^2 - z^2}} - \frac{m^2}{z\sqrt{m^2 - z^2}}$$

$$\doteq -\frac{1}{z}\sqrt{m^2 - z^2} \tag{18}$$

In each of these expressions the first term is neglected in the final form since it is small when the inequalities on the left are considered. The substitution of these results in (14) yields:

$$k_4b > m: \quad \frac{H_{m-1}^{(1)}(k_4b)}{H_m^{(1)}(k_4b)} \doteq \frac{m + i\sqrt{(k_4b)^2 - m^2}}{k_4b} \tag{19}$$

$$k_4b < m: \quad \frac{H_{m-1}^{(1)}(k_4b)}{H_m^{(1)}(k_4b)} \doteq \frac{m - \sqrt{m^2 - (k_4b)^2}}{k_4b} \tag{20}$$

With (19) and (20), (12) becomes:

$$\Delta_0 = \lim_{\substack{b\to\infty \\ D\to\infty}} \left\{ i + 2 \sum_{m=2}^{k_4 b} \left(\frac{b-d}{b}\right)^{2m} \left[\frac{m + i\sqrt{(k_4 b)^2 - m^2}}{k_4 b}\right] \right.$$

$$\left. + 2 \sum_{k_4 b}^{\infty} \left(\frac{b-d}{b}\right)^{2m} \left[\frac{m - \sqrt{m^2 - (k_4 b)^2}}{k_4 b}\right] \right\} \tag{21}$$

The sums in this expression can be converted into integrals with the substitutions $x = m/k_4 b$, $dx = 1/k_4 b$. Also,

$$(1 - d/b)^{2m} \doteq \exp(-2md/b) = \exp(-2mk_4 d/k_4 b) = \exp(-2xk_4 d) \tag{22}$$

$$\frac{m + i\sqrt{(k_4 b)^2 - m^2}}{k_4 b} = x + i\sqrt{1 - x^2} \tag{23}$$

$$\frac{m - \sqrt{m^2 - (k_4 b)^2}}{k_4 b} = x - \sqrt{x^2 - 1} \tag{24}$$

With these substitutions, (21) is approximated by:

$$\Delta_0 = i + 2k_4 b \int_0^1 (x + i\sqrt{1 - x^2}) e^{-Ax}\, dx + 2k_4 b \int_1^\infty (x - \sqrt{x^2 - 1}) e^{-Ax}\, dx \tag{25}$$

where $A \equiv 2k_4 d$. This expression can be rearranged as follows:

$$\Delta_0 = i + 2k_4 b \left[\int_0^\infty x e^{-Ax}\, dx - \int_1^\infty \sqrt{x^2 - 1}\, e^{-Ax}\, dx + i \int_0^1 \sqrt{1 - x^2}\, e^{-Ax}\, dx \right] \tag{26}$$

The several integrals in (26) are evaluated in the Appendix. The final expression for Δ_0 is obtained with (A-17). It is

$$(\Delta_0/k_4 b) = 2\left[\frac{1}{A^2} - \frac{K_1(A)}{A} + \frac{i\pi I_1(A)}{2A} - i\left(\frac{A}{3} + \frac{A^3}{45} + \frac{A^5}{1575} + \frac{A^7}{99225} + \dots\right)\right] \tag{27}$$

where $A = 2k_4 d$, with d the distance from the axis of the antenna in air to

the plane boundary between the air and the material half-space. Note that in the limit $b \to \infty$, the first term in (26), i, vanishes.

The substitution of (9) and (27) into (4) gives

$$k_L = k_2 \left\{ 1 + \frac{2}{\ln(2d/a)} \left[\frac{1}{(2k_4 d)^2} - \frac{K_1(2k_4 d)}{2k_4 d} + \frac{i\pi I_1(2k_4 d)}{4k_4 d} \right.\right.$$

$$\left.\left. - i\left(\frac{2k_4 d}{3} + \frac{(2k_4 d)^3}{45} + \frac{(2k_4 d)^5}{1575} + \ldots \right) \right] \right\}^{1/2} \tag{28}$$

Thus, the complex wave number k_L for the antenna in air over a plane medium with characteristic wave number k_4 depends only on the distance d from the axis of the antenna to the plane boundary, the radius a of the antenna and the wave numbers k_4 and $k_2 = k_0$ of the air. Note that if $|2k_4 d| \leq 1$, only two terms in the series in (28) are required. For example, when k_4 is real and $k_4 d = 0.5$, $K_1(1) = 0.6019$, $I_1(1) = 0.5652$ and

$$k_L = k_2 \left\{ 1 + \frac{2}{\ln(2d/a)} [1 - 0.6019 + i0.8878 - i(0.3333 + 0.0222)] \right\}^{1/2}$$

$$= k_2 \left\{ 1 + \frac{0.80 + i1.06}{\ln(2d/a)} \right\}^{1/2} \tag{29}$$

With the complex wave number k_L determined, the characteristic impedance is obtained from (3) with (9) and (28). Thus,

$$Z_c = \frac{k_L \zeta_2}{2\pi k_2} \cosh^{-1} \frac{d}{a} \doteq \frac{k_L \zeta_2}{2\pi k_2} \ln \frac{2d}{a} \tag{30}$$

The distribution of current along the antenna is given by (2), the driving-point admittance by (8), with k_L and Z_c defined in (28) and (30).

The effective line constants per unit length are readily evaluated since $k_L^2 = -y_L z_L$ and $Z_c^2 = z_L/y_L$. Thus,

$$y_L = g_L - ib_L = -ik_L/Z_c = -\frac{i2\pi\omega\tilde{\epsilon}_2}{\ln(2d/a)} \tag{31}$$

so that with $b_L = \omega c_L$,

$$g_L = \frac{2\pi\sigma_{2e}}{\ln(2d/a)} \quad , \quad c_L = \frac{2\pi\varepsilon_{2e}}{\ln(2d/a)} \tag{32}$$

where σ_{2e} and ε_{2e} are the real effective conductivity and permittivity of the insulating cylinder. Similarly,

$$z_L = r_L - ix_L = -ik_L Z_c = -\frac{ik_L\zeta_2}{2\pi k_2} \ln\frac{2d}{a} \tag{33}$$

With (28) and $k_2\zeta_2 = \omega\mu$,

$$z_L = z^e + z_4^i = -i\omega\ell^e + z_4^i \tag{34}$$

with

$$\ell^e = \frac{\mu}{2\pi} \ln\frac{2d}{a} \tag{35}$$

$$z_4^i = \frac{-i\omega\mu}{\pi}\left[\frac{1}{(2k_4d)^2} - \frac{K_1(2k_4d)}{2k_4d} + \frac{i\pi I_1(2k_4d)}{4k_4d} - i\left(\frac{2k_4d}{3} + \frac{(2k_4d)^3}{45} + \frac{(2k_4d)^5}{1575} + \ldots\right)\right] \tag{36}$$

Thus, the antenna over a half-space characterized by $|k_4| >> |k_2|$ behaves like a transmission line with the line constants given in (32), (35) and (36). If the ohmic losses in the antenna are to be included, an additional impedance per unit length, viz.,

$$z_1^i = \frac{\sqrt{-i\omega\mu_1/\sigma_1}}{2\pi a} \tag{37}$$

must be added to z_L in (34) and included in $k_L^2 = -z_Ly_L$ and $Z_c = \sqrt{z_L/y_L}$. In (37), μ_1 and σ_1 are the permeability and conductivity of the metal forming the antenna.

4. THE AXIAL ELECTRIC FIELD ON THE SURFACE OF THE HALF-SPACE

The transform of the axial component of the electric field at a radius r from the axis of the insulating region 2 that contains the eccentrically located antenna is [10]:

$$\bar{E}_z(r,\theta) = \frac{i\bar{I}}{2\pi\omega\tilde{\varepsilon}_2}\left[(k_2^2 - \zeta^2)\Omega(r,\theta) + \frac{k_2^2}{k_4 b}\Delta(r,\theta)\right] \tag{38}$$

where θ is the angle measured from the radius through the antenna where this is closest to the boundary, $\bar{I}$ is the transform of the current, and

$$\Delta(r,\theta) = \frac{H_0^{(1)}(k_4b)}{H_1^{(1)}(k_4b)} + 2\sum_{m=1}^{\infty}\left(\frac{rx_0}{b^2}\right)^m \frac{H_m^{(1)}(k_4b)}{H_{m+1}^{(1)}(k_4b)} \cos m\theta \tag{39}$$

$$\Omega(r,\theta) = \frac{1}{2}\ln\left[\frac{(rx_0)^2 - 2b^2 rx_0 \cos\theta + b^4}{b^2(r^2 - 2x_0 r\cos\theta + x_0^2)}\right] \tag{40}$$

At the boundary $r = b$ and with (10), $x_0 \to b - d$ as $b \to \infty$. Hence,

$$\Omega(b,\theta) = 0 \tag{41}$$

$$\Delta(b,\theta) = \frac{H_0^{(1)}(k_4b)}{H_1^{(1)}(k_4b)} + 2\sum_{m=1}^{\infty}\left(\frac{b-d}{b}\right)^m \frac{H_m^{(1)}(k_4b)}{H_{m+1}^{(1)}(k_4b)} \cos m\theta \tag{42}$$

It follows that

$$\bar{E}_z(b,\theta) \doteq \frac{i\omega\mu\bar{I}}{2\pi}\frac{\Delta(b,\theta)}{k_4b} \tag{43}$$

which is independent of the transform variable ζ. Hence, the inverse Fourier transform with respect to z gives simply:

$$E_z(b,\theta) = \frac{i\omega\mu I}{\pi}\frac{\Delta(b,\theta)}{k_4b} \tag{44}$$

for the z-component of the electric field in terms of the current I.

The evaluation of $\Delta(b,\theta)/k_4b$ can be carried out by the procedure used for Δ_0/k_4b. With the notation $L = b\theta$ (where L is the distance along the boundary from $\theta = 0$ in directions at right angles to the antenna), $B = k_4L$ and $A = k_4d$ (not $2k_4d$ as in the evaluation of Δ_0/k_4b), the equation corresponding to (25) is

$$\Delta(b,\theta) = i + 2k_4b\left[\int_0^1 (x + i\sqrt{1-x^2})e^{-Ax}\cos Bx\,dx + \int_1^{\infty}(x - \sqrt{x^2-1})e^{-Ax} \times \cos Bx\,dx\right] \tag{45}$$

(Note that this reduces to (25) when $B = 0$.) The evaluation of the square bracket in (45) is carried out in the Appendix, and given in its general form in (A-13). In the limit $b \to \infty$, the first term in (45) vanishes. The final result is

$$\lim_{\substack{b\to\infty \\ D\to\infty}} \frac{\Delta(b,\theta)}{k_4 b} = \frac{\Delta(d,L)}{k_4 b} = 2\left\{\frac{A^2 - B^2}{(A^2 + B^2)^2} - \frac{K_1(A - iB)}{A - iB} + i\,\mathrm{Im}\,\frac{\pi}{2(B + iA)} \times [E_1(B + iA) + Y_1(B + iA) - \frac{2}{\pi}]\right\} \tag{46}$$

where K_1 is the modified Bessel function of the second kind and order one, E_1 is the Weber function of order one and Y_1 is the Bessel function of the second kind and order one. These are defined in the Appendix. Accordingly, with $A = k_4 d$, $B = k_4 L$,

$$E_z(d,L) = \frac{i\omega\mu I}{\pi}\left\{\frac{A^2 - B^2}{(A^2 + B^2)^2} - \frac{K_1(A - iB)}{A - iB} + i\,\mathrm{Im}\,\frac{\pi}{2(B + iA)}\,[E_1(B + iA) + Y_1(B + iA) - \frac{2}{\pi}]\right\} \tag{47}$$

At $L = 0$, (A-17) gives:

$$E_z(d,0) = \frac{i\omega\mu I}{\pi}\left\{\frac{1}{A^2} - \frac{K_1(A)}{A} + \frac{i\pi I_1(A)}{2A} - i\left[\frac{A}{3} + \frac{A^3}{45} + \frac{A^5}{1575} + \dots\right]\right\} \tag{48}$$

This is the axial field at the boundary surface along the normal projection of the antenna as shown in Fig. 4.

Since these expressions are not particularly transparent, it is advantageous to examine them in certain limiting cases. Consider first their forms for large arguments, i.e., when $|A^2 + B^2| >> 1$. The applicable formula is (A-19). Thus,

$$E_z(d,L) = \frac{i\omega\mu I}{\pi}\left\{\frac{A^2 - B^2}{(A^2 + B^2)^2} - \frac{1}{A - iB}\sqrt{\frac{\pi}{2(A - iB)}}\, e^{-(A - iB)}\left[1 + \frac{3}{8(A - iB)} - \frac{15}{128(A - iB)^2} + \dots\right] \mp i\,\mathrm{Im}\,\frac{1}{B + iA}\left[1 + \frac{1}{(B + iA)^2} - \frac{3}{(B + iA)^4}\right.\right.$$

(cont.)

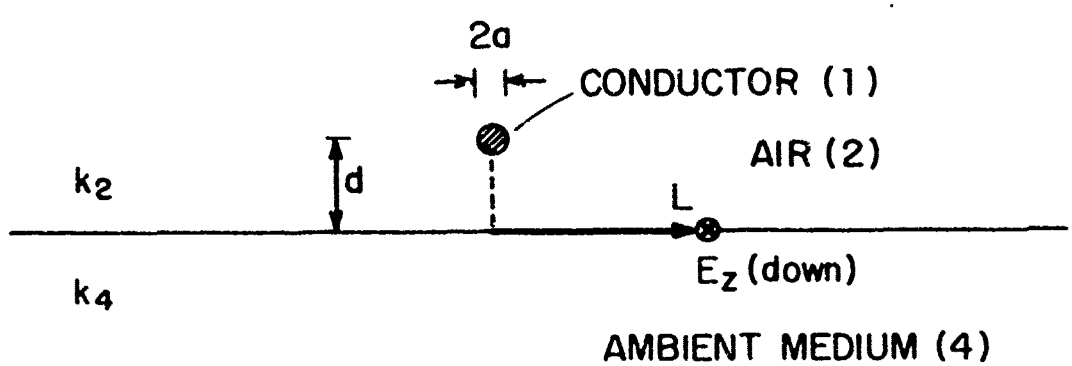

FIG. 4 HORIZONTAL CONDUCTOR (1) IN AIR (2) OVER AMBIENT MEDIUM (4); E_z AT BOUNDARY

$$+ \frac{45}{(B + iA)^6} - \cdots \Big] \Big\} \tag{49}$$

The field at $L = 0$ for $|A| >> 1$ is obtained with (A-20). It is

$$E_z(d,0) = \frac{i\omega\mu I}{\pi} \left\{ \frac{1}{A^2} - \frac{1}{A} \sqrt{\frac{\pi}{2A}}\, e^{-A} \left[1 + \frac{3}{8A} - \frac{15}{128A^2} + \cdots \right] + \frac{i}{A} \left[1 - \frac{1}{A^2} - \frac{3}{A^4} - \cdots \right] \right\} \tag{50}$$

The condition $|A^2 + B^2| = |k_4^2(L^2 + d^2)| >> 1$ can be satisfied in several ways. When $|k_4|$ is made very large - or infinite for a perfect conductor - the field along the surface of the half-space is very small. For $k_4 \to \infty$,

$$E_z(d,L) = 0 \tag{51}$$

as it should. When $|A| = |k_4 d|$ is very large because $|k_4|$ is large, as for a good conductor, $E_z(d,L)$ is very small. For a given, sufficiently large $k_4 d$, the electric field along the boundary decreases with increasing $B = k_4 L$ according to (49). When $B^2 >> A^2$, the field is obtained with (A-21). It is

$$E_z(d,L) \doteq -\frac{i\omega\mu I}{\pi} \left\{ \frac{1}{B^2} + \frac{i}{B} \sqrt{\frac{i\pi}{2B}}\, e^{iB} \left[1 + \frac{3i}{8B} + \frac{15}{128B^2} + \cdots \right] \right\} \tag{52}$$

Thus, for large values of $B = k_4 L$, the field decays radially outward along the boundary surface as $1/(k_4 L)^{3/2}$.

The condition for small arguments $|k_4^2(L^2 + d^2)| = |A^2 + B^2| << 1$ is quite restrictive but not physically unavailable. Note that the conditions $|k_4| >> |k_2|$ and $|k_2 d| << 1$ have been imposed from the outset. To require, in addition, that $|k_4^2 d^2| << 1$ implies that $|k_2 d| <<< 1$. Specifically, if k_4 applies to lake water with $\epsilon_r = 81$, it is given by $k_4 \doteq 9\omega/c = 3\omega \times 10^{-8}\ m^{-1}$. If the antenna is located at a distance of $d = 1$ m from the surface of the water, $k_4 d = 3\omega \times 10^{-8}$. For this to be small compared with one, viz., $|k_4 d| \leq 0.1$, ω must not exceed about 3×10^6. Note that $k_2 d = \omega d/c \leq 0.01$. Similarly, for sea water with $\sigma = 4$ mho/m, $k_4 = (i\omega\mu\sigma)^{1/2}$ and $|k_4| = 4(\omega\pi \times 10^{-7})^{1/2}$. With $d = 1$ m, $|k_4 d| = 4(\omega\pi \times 10^{-7})^{1/2}$. This will satisfy the condition $|k_4 d| \leq 0.1$ if ω does not exceed about 2×10^3. At this frequency $k_2 d = 6.67 \times 10^{-6}$, $k_4 d = 0.1e^{i\pi/4}$.

Subject to the condition $|A^2 + B^2| << 1$, an approximate formula for

$E_z(d,L)$ is obtained with (A-27). It is

$$E_z(d,L) \sim \frac{i\omega\mu I}{2\pi}\left[\ln\frac{2}{k_4\sqrt{L^2+d^2}} - \gamma + \frac{1}{2} + i\,\frac{\pi}{2}\right] \tag{53}$$

It is interesting to compare this expression with the incident field generated by the current in the wire if this is treated as a line source. It is

$$E_z^{inc}(d,L) = i\omega\mu I\,\frac{i}{4}\,H_0^{(1)}(k_2\sqrt{L^2+d^2}) \tag{54}$$

Sufficiently near the antenna where $k_2^2(L^2 + d^2) << 1$, the small-argument form of the Hankel function is a satisfactory approximation. This gives:

$$E_z^{inc} \sim \frac{i\omega\mu I}{2\pi}\left[\ln\frac{2}{k_2\sqrt{L^2+d^2}} - \gamma + i\,\frac{\pi}{2}\right] \tag{55}$$

The total field $E_z(d,L)$ at the boundary as given in (53) can be expressed as the sum of the incident field plus the field scattered from the matter-filled half-space. With (53) and (55) it is

$$E_z(d,L) = E_z^{inc}(d,L) + E_z^{scat} = \frac{i\omega\mu I}{2\pi}\left[\ln\frac{2}{k_2\sqrt{L^2+d^2}} - \gamma + i\,\frac{\pi}{2} - \ln\frac{k_4}{k_2} + \frac{1}{2}\right] \tag{56}$$

This formula indicates that at least within the range of the conditions $|k_4|^2 >> |k_2|^2$ and $|k_4d| << 1$, the contribution of the currents and charges in the matter-filled half-space to $E_z(d,L)$ on its surface is the scattered field

$$E_z^{scat} = -\frac{i\omega\mu I}{2\pi}\left[\ln\frac{k_4}{k_2} - \frac{1}{2}\right] \tag{57}$$

This field cannot be interpretted as being due to an image current in an image antenna at a distance 2d from the antenna since it is independent of d.

For lake water with $\epsilon_r \doteq 80$, d = 1 m, $\omega = 3 \times 10^6$, $k_2d = 0.01$, $k_4d = 0.09$, the ratio of the scattered to the incident field is:

$$\frac{E^{scat}(d,0)}{E^{inc}(d,0)} = - \frac{\ell n(k_4/k_2) - 0.5}{\ell n(2/k_2 d) - \gamma + i\pi/2} = 0.35e^{i2.82} = 0.35e^{i162^\circ} \tag{58}$$

The ratio of the total field to the incident field is:

$$\frac{E(d,0)}{E^{inc}(d,0)} = \frac{E^{scat}(d,0)}{E^{inc}(d,0)} + 1 = 0.68e^{i0.16} = 0.68e^{i9^\circ.5} \tag{59}$$

For sea water with $\sigma = 4$ mho/m, $d = 1$ m, $\omega = 2 \times 10^3$, $k_2 d = 6.67 \times 10^{-6}$, $k_4 d = 0.1e^{i\pi/4}$, so that

$$\frac{E^{scat}(d,0)}{E^{inc}(d,0)} = 0.75e^{i3.09} = 0.75e^{i177^\circ} \tag{60}$$

and

$$\frac{E(d,0)}{E^{inc}(d,0)} = 0.25e^{i0.16} = 0.25e^{i9^\circ} \tag{61}$$

It is seen that the effect of the dielectric (lake water) like that of the conductor (sea water) is to combine with the incident field to produce a total field at the boundary that is substantially smaller than the incident field. With lake water, the incident and scattered fields differ in phase by 162°; with sea water the phase difference is about 177° for the particular distance d and frequency.

5. THE GENERALIZED BEVERAGE ANTENNA; CONCLUSION

The well-known Beverage antenna [1]-[4] consists of a wire up to several wavelengths long that is stretched parallel to and an electrically short distance from the surface of the earth to which it is connected at both ends through suitable resistive terminations. Such an arrangement to achieve a traveling wave is possible only when the earth is sufficiently conducting to provide an adequate ground connection. A more generally useful method of obtaining a traveling wave along a sufficiently long wire stretched parallel to a material half-space of any material with properties that range from those of a good dielectric (lake water) to those of a good conductor (sea water) is to connect the resistive terminations in series with the wire at distances near a quarter wavelength from the ends as in Fig. 1c. An analysis of this

generalized form of the Beverage antenna in the framework of the present theory is reserved for another report.

In this report a new approach has been given for the traveling wave antenna over an imperfectly conducting or dielectric half-space. The complex wave number, the distribution of current, and the electric field tangent to the boundary have been determined. The problem of calculating the radiation field of such an antenna with the known distribution of current can be accomplished using well known methods.

APPENDIX: EVALUATION OF INTEGRALS

The following integrals occur in (26) with $B = 0$ and $A = 2k_4 d$ and in (45) with $B = k_4 L$ and $A = k_4 d$:

$$I = \int_0^1 (x + i\sqrt{1 - x^2})e^{-Ax} \cos Bx \, dx + \int_1^\infty (x - \sqrt{x^2 - 1})e^{-Ax} \cos Bx \, dx \tag{A-1}$$

This expression can be rearranged as follows:

$$I = \int_0^\infty xe^{-Ax} \cos Bx \, dx + \frac{1}{2}\left[\int_0^1 i\sqrt{1 - x^2}\, e^{-(A - iB)x} \, dx + \int_1^\infty (\sqrt{x^2 - 1}\, e^{-(A - iB)x} \, dx\right.$$

$$- \int_1^\infty \sqrt{x^2 - 1}\, e^{-(A - iB)x} \, dx + \frac{1}{2}\left[\int_0^1 i\sqrt{1 - x^2}\, e^{-(A + iB)x} \, dx\right.$$

$$\left. - \int_1^\infty \sqrt{x^2 - 1}\, e^{-(A + iB)x} \, dx\right] \tag{A-2}$$

Here the first integral is:

$$I_1 = \int_0^\infty xe^{-Ax} \cos Bx \, dx = \frac{A^2 - B^2}{(A^2 + B^2)^2} \tag{A-3}$$

With the substitution $x = \cosh\theta$, the fourth integral is evaluated as follows:

$$I_2 = \int_1^\infty \sqrt{x^2 - 1}\, e^{-(A - iB)x} \, dx = \int_0^\infty \sinh^2\theta\, e^{-(A - iB)\cosh\theta} \, d\theta \equiv \frac{K_1(A - iB)}{A - iB} \tag{A-4}$$

where $K_1(z)$ is the modified Bessel function of the first order and second kind. The remaining integrals are equivalent to

$$I_3 = \frac{1}{2}\left[\int_0^1 i\sqrt{1 - x^2}\, e^{-(A - iB)x} \, dx + \int_1^\infty \sqrt{x^2 - 1}\, e^{-(A - iB)x} \, dx\right] - \left[\int_0^1 i\sqrt{1 - x^2}\right.$$

$$\left. \times e^{-(A - iB)x} \, dx + \int_1^\infty \sqrt{x^2 - 1}\, e^{-(A - iB)x} \, dx\right]^* \tag{A-5}$$

where the asterisk denotes the complex conjugate of the expression in the

brackets. Since the second square bracket in (A-5) is the complex conjugate of the first, their difference is twice the imaginary part of the first. That is,

$$I_3 = i\mathrm{Im}\left[\int_0^1 i\sqrt{1-x^2}\, e^{-(A-iB)x}\,dx + \int_1^\infty \sqrt{x^2-1}\, e^{-(A-iB)x}\,dx\right] \qquad \text{(A-6)}$$

This is an integral along the real axis from zero to infinity with an upward deflection around the point $x = 1$ and the attached downward branch cut. The path of integration can be rotated counterclockwise 90° to the y-axis with the substitution $x = iy$. This leaves simply

$$I_3 = i\mathrm{Im}\int_0^\infty i\sqrt{1+y^2}\, e^{-(A-iB)iy}\, i\,dy = -i\mathrm{Im}\int_0^\infty \cosh^2\theta\, e^{-(B+iA)\sinh\theta}\,d\theta \qquad \text{(A-7)}$$

The last integral in (A-7) is readily identified as the sum of the following integrals with suitable coefficient:

$$E_0(z) + Y_0(z) = -\frac{2}{\pi}\int_0^\infty e^{-z\sinh\theta}\,d\theta \qquad \text{(A-8)}$$

$$E_2(z) + Y_2(z) = -\frac{2}{\pi}\int_0^\infty \cosh 2\theta\, e^{-z\sinh\theta}\,d\theta \qquad \text{(A-9)}$$

$E_n(z)$ is the Weber function of order n [11, p. 35], [12], $Y_n(z)$ is the Bessel function of the second kind and order n. With the well-known recurrence relations

$$Y_0(z) + Y_2(z) = \frac{2}{z}Y_1(z) \qquad \text{(A-10)}$$

$$E_0(z) + E_2(z) = \frac{2}{z}\left[E_1(z) - \frac{2}{\pi}\right] \qquad \text{(A-11)}$$

(A-7) becomes

$$I_3 = i\mathrm{Im}\frac{\pi}{2(B+iA)}\left[E_1(B+iA) + Y_1(B+iA) - \frac{2}{\pi}\right] \qquad \text{(A-12)}$$

When (A-3), (A-4) and (A-12) are combined to give (A-2), this becomes

$$I = I_1 - I_2 + I_3$$

$$I = \frac{A^2 - B^2}{(A^2 + B^2)^2} - \frac{K_1(A - iB)}{A - iB} + i\,\mathrm{Im}\,\frac{\pi}{2(B + iA)}\left[E_1(B + iA) + Y_1(B + iA) - \frac{2}{\pi}\right] \tag{A-13}$$

Special Case with B = 0:

An important special case is when $B = 0$. In this case

$$I = \frac{1}{A^2} - \frac{K_1(A)}{A} + \frac{i\pi I_1(A)}{2A} - \frac{i\pi}{2A}\left[E_1(iA) - \frac{2}{\pi}\right] \tag{A-14}$$

where $I_1(A)$ is the modified Bessel function of the first kind and first order. This follows with

$$\mathrm{Im}\,\frac{Y_1(iA)}{iA} = \mathrm{Im}\,\frac{1}{A}\,[-H_1^{(1)}(iA) + J_1(iA)] = \mathrm{Im}\,\frac{1}{A}\left[\frac{2}{\pi}K_1(A) + iI_1(A)\right] = \frac{I_1(A)}{A} \tag{A-15}$$

and [11, p. 35]

$$E_1(z) = \frac{2}{\pi}\left[1 - \frac{z^2}{3} + \frac{z^4}{45} - \frac{z^6}{1575} + \frac{z^8}{99225} - \dots\right] \tag{A-16}$$

Note that $K_1(A)$ is real. Since the modified Bessel functions of real arguments are available in Tables, a convenient form of (A-14) is:

$$I = \frac{1}{A^2} - \frac{K_1(A)}{A} + \frac{i\pi I_1(A)}{2A} - i\left[\frac{A}{3} + \frac{A^3}{45} + \frac{A^5}{1575} + \frac{A^7}{99225} + \dots\right] \tag{A-17}$$

It is this form that is used to obtain (27) from (26).

Special Case for Large Arguments: $A^2 + B^2 >> 1$:

When the argument is large, the following series is useful [12]:

$$E_1(z) + Y_1(z) \sim -\frac{2}{\pi z^2}\left[1 - \frac{3}{z^2} + \frac{45}{z^4} - \dots\right] \tag{A-18}$$

With (A-18) and the usual expressions for Bessel functions with large argument, it follows that

$$I = \frac{A^2 - B^2}{(A^2 + B^2)^2} - \frac{1}{A - iB}\sqrt{\frac{\pi}{2(A - iB)}}\, e^{-(A - iB)}\left[1 + \frac{3}{8(A - iB)} - \frac{15}{128(A - iB)^2} + \cdots\right]$$

$$- i\mathrm{Im}\,\frac{1}{B + iA}\left[1 + \frac{1}{(B + iA)^2} - \frac{3}{(B + iA)^4} + \frac{45}{(B + iA)^6} - \cdots\right] \qquad \text{(A-19)}$$

When $|A| >> 1$, $B = 0$,

$$I = \frac{1}{A^2} - \frac{1}{A}\sqrt{\frac{\pi}{2A}}\, e^{-A}\left[1 + \frac{3}{8A} - \frac{15}{128A^3} + \cdots\right] + \frac{i}{A}\left[1 - \frac{1}{A^2} - \frac{3}{A^4} - \cdots\right] \qquad \text{(A-20)}$$

On the other hand, if $|B| >> 1$ and $|B| >> |A|$,

$$I = -\frac{1}{B^2} - \frac{i}{B}\sqrt{\frac{i\pi}{2B}}\, e^{iB}\left[1 + \frac{3i}{8B} + \frac{15}{128B^3} + \cdots\right] \qquad \text{(A-21)}$$

Special Case for Small Arguments: $|B^2 + A^2| << 1$:

With small arguments [13]

$$K_1(z) \sim \frac{z}{2}\ln\frac{z}{2} + \frac{1}{z} + \frac{z}{4}(2\gamma - 1) \qquad \text{(A-22)}$$

$$\pi Y_1(z) \sim 2(\gamma + \ln\frac{z}{2}) - \frac{2}{z} - \frac{z}{2} \cdots \qquad \text{(A-23)}$$

so that with (A-16)

$$\frac{\pi}{2z}\left[Y_1(z) + E_1(z) - \frac{2}{\pi}\right] \sim \frac{1}{2}\left(\gamma + \ln\frac{z}{2} - \frac{1}{2}\right) - \frac{1}{z^2} - \frac{z}{3} \qquad \text{(A-24)}$$

The use of (A-21)-(A-23) in (A-13) gives:

$$I \sim \frac{A^2 - B^2}{(A^2 + B^2)^2} - \frac{1}{(A - iB)^2} - \frac{1}{2}\left[\ln\frac{A - iB}{2} + \gamma - \frac{1}{2}\right] + i\mathrm{Im}\left[\frac{1}{2}\left(\gamma + \ln\frac{B + iA}{2} - \frac{1}{2}\right)\right.$$

$$\left. - \frac{1}{(B + iA)^2} - \frac{B + iA}{3}\right] \qquad \text{(A-25)}$$

This reduces to:

$$I \sim -\frac{1}{2} \ln \frac{\sqrt{A^2 + B^2}}{2} - \frac{1}{2}\left(\gamma - \frac{1}{2}\right) + i\left(\frac{\pi}{4} - \frac{A}{3}\right) \tag{A-26}$$

Here the leading real and imaginary terms are

$$I \sim \frac{1}{2}\left[\ln \frac{2}{\sqrt{A^2 + B^2}} - \gamma + \frac{1}{2} + i\,\frac{\pi}{2}\right] \tag{A-27}$$

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